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Preschoolers' Performance on Simple Two- and Three-Term Arithmetic Problems: Working Memory Constraints and Conceptual Understanding

BY

JULIETTE S. KLEIN ©

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment

of the requirements for the degree

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled "Preschoolers' Performance on Simple 2- and 3-Term Arithmetic Problems: Working Memory Constraints but Conceptual Understanding" submitted by Juliette S. Klein in partial fulfillment of the requirements for the degree of Master of Science.

Abstract

The first goal of the research was to investigate what makes simple two-term arithmetic difficult for preschoolers. To reduce task demands, we presented 4-year-olds with a nonverbal task described by Huttenlocher et al. (1994) in which the children solved two-term addition and subtraction problems using counters. Using regression analyses, we found that the best predictor of errors was the Representational Set Size of the problem. This suggests that errors may occur as a result of the children having difficulty representing larger numbers in short-term memory.

The second goal of the research was to investigate whether preschoolers have a conceptual understanding of the inversion relation between addition and subtraction. Using a nonverbal method again, preschoolers solved relatively novel three-term problems of the form $\underline{a}+\underline{b}-\underline{c}$ and $\underline{a}+\underline{b}-\underline{b}$ and their solution procedures were recorded. Children showed behaviours consistent with using conceptual shortcuts based on the principles of associativity and inversion.

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Preschoolers' Performance on Simple Two- and Three-Term Arithmetic Problems: Working Memory Constraints and Conceptual Understanding

Introduction

Performance on simple arithmetic problems has been studied often in order to make inferences about some of children's cognitive processes. Wynn (1992) found evidence to suggest that even infants (mean age 5 months) are capable of performing simple addition and subtraction. The ability to enumerate sets of objects increases as children grow older. Klein (1984, cited in Klein & Starkey, 1987) found that young children (4- to 6-year-olds) counted and used one-to-one correspondences to construct different numerical representations of arithmetic problems. Gelman and Gallistel (1978) proposed that children's counting is conceptually based and that counting is a fundamental skill that is built upon in the later preschool years as they begin to add and subtract.

Other researchers have found that although preschoolers have some ability to add and subtract small numbers, they are not very proficient, especially as the operands in the problems get larger (e.g., Levine, Jordan, & Huttenlocher, 1992). It is important to note, however, that when 4-year-olds make errors on addition or subtraction problems, it is most usually an error in the correct direction; that is, children do not subtract when they should be adding (Ginsburg, Klein, & Starkey, 1998). Thus preschoolers understand that adding makes a set bigger, whereas subtracting makes a set smaller. Preschoolers make many errors when solving simple arithmetic problems, however, and there is little research on what makes some problems more difficult than others for these children.

Although there are many potential reasons why preschoolers have difficulties with simple arithmetic, three particular reasons are focussed on with this research: nonmathematical task demands, memory load, and knowledge of arithmetic principles. First, young children may find simple arithmetic problems difficult to solve because the nonmathematical demands of the task are too high. Levine et al. (1992) reported that 4-, 5-, and 6-year-olds found story and symbolic number arithmetic problems difficult, even with very small numbers, but had less trouble when the problems were presented nonverbally. In a non-verbal condition, the researchers presented an array of counters to represent different numerosities to the children and then asked the children to construct an identical array. After the children had constructed their array, the researcher's array was concealed. Children then watched as other counters were either added to or removed from the concealed array. The children were then asked to make his or her array match the researcher's concealed array. The researchers found that children were more likely to construct correct arrays when they were not forced to concentrate on verbalizing their answers. This nonverbal method of presenting arithmetic tasks to children has been used with success in several studies (e.g., Huttenlocher, Jordan, & Levine, 1994) and can be adopted to explore other sources of difficulty that preschoolers might have when solving arithmetic problems.

In the present study this nonverbal method was used to investigate two other hypotheses about preschoolers' difficulties in simple arithmetic. The first hypothesis is that young children's working memory is not fully mature and so becomes overloaded and fails easily. Performance on two-term addition and subtraction problems (e.g., 2+3) was analyzed to determine whether the construct of working memory capacity could be

used to explain the 4-year-olds' errors. The second hypothesis is that preschoolers lack the conceptual understanding of arithmetic necessary for effective solution. Performance on relatively novel three-term problems (e.g., 2+1-1) was analyzed to determine whether the children would spontaneously use conceptual shortcuts to solve the problems more quickly and easily.

Difficulty of Simple Two-Term Arithmetic Problems

Preschool children tend to make more errors as problem size increases (Huttenlocher et al., 1994). This problem-size effect also appears when older children and adults are presented with arithmetic problems (e.g., Ashcraft, 1990, LeFevre, Sadesky, & Bisanz, 1996). Problem size can be indexed by such structural elements as minimum or maximum operand or by the sum or difference of the problem.

Levine et al. (1992) found that preschoolers can solve some small addition and subtraction problems (e.g., 1+2), but that they were more likely to make errors as the operands of the problem got larger (e.g., as the first term approaches 4). There are at least two reasons why children may be making errors on the arithmetic problems. First, children might not understand the concepts of addition and subtraction. That is, they can solve very small problems but they cannot apply those concepts when the addends are larger than 1 or 2. The other possible explanation is that children's working memory fails them when they try to solve problems with larger addends. That is, children use all their working memory resources trying to remember the addends of the problem and, as a result, they have no resources left over to solve the problem correctly.

Preschool children might show a problem-size effect for several reasons. They may have difficulty counting larger numbers, or they may have trouble with the actual

calculations involved in finding the answer to the problem. They might also have difficulties with larger problems if representing the operands, sums, or differences of the problems exceed short-term memory demands. In this study I investigated the hypothesis that a substantial portion of young children's difficulties in simple arithmetic can be traced to working memory demands and limitations.

Many researchers have studied working memory and have developed different theories about it. For example, Pascual-Leone (1970) suggested that there are a defined number of chunks of information that can be held in working memory at any one time. Adults' superior performance on working memory tasks is explained by the fact that they can hold more chunks of information than can young children. Taking a somewhat different approach, Chi (1976) proposed that as children get older, they develop more efficient strategies for holding information in memory. Because they can keep information more easily, they are therefore able to hold more chunks of information. Baddeley (1996) developed a model of working memory that consists of separate storage capacities for verbal and spatial information, and that a third component serves as a central executive processor, coordinating the other two components. Baddeley claims that changes in working memory occur due to increased capacity in the two storage components and increases in the competency of the central executive processor.

Working memory has been studied extensively with many different cognitive tasks. Many people have studied digit span or word span in an attempt to document how much information can be held in working memory at once (e.g., Kail, 1997; Kail & Park, 1994).

Working memory also has been studied using many arithmetic tasks. For example, Adams and Hitch (1997) investigated whether young children's arithmetic ability was influenced more by working memory or conceptual constraints. They presented children with arithmetic problems that were either verbal or had a visual cue to help them remember the operands of the problem. They found that children were more likely to solve the problem correctly when the operands were visible. This suggests that when children did not have to hold the operands in memory, they could use more of their working memory resources to solve the problems.

To investigate the role of working memory in young children's arithmetic, certain assumptions appear to be reasonable. Presumably children either retrieve answers from memory, or solve by counting chips and calculating. Although retrieval usually does not place large demands on working memory (Siegler, 1998), calculating the answer does. These requirements may or may not exceed the capacity of working memory (e.g., Adams & Hitch, 1997). Case, Kurland, and Goldberg (1982) argue that information is stored in working memory as "chunks", and that at any given age a child can only hold a certain number of chunks in memory. When task demands exceed the number of chunks available in working memory, the result is an increased probability of the child failing to solve the problem because the child has no working memory resources remaining to calculate the answer.

Because the kindergarten children from my previous study (Klein, 1996) used a correct calculation strategy on 72% of the problems, I assumed that the preschoolers in the present study also would use calculation solutions (rather than retrieval) on a majority of the problems. All the problems were in form of $\underline{a}+\underline{b}$ or $\underline{a}-\underline{b}$, where $\underline{a}$ represents the first

term of the problem, and b represents the second term. The model I propose has several general features irrespective of whether the children are adding or subtracting overtly. I assume that the counters the child sees are enumerated and coded internally in working memory in a set, S, that can be incremented and decremented. S is limited by the storage capacity available in the child's working memory.

In this study, each counter represented in S is assumed to require one chunk of information in S. Figure 1 represents a model of how the children might solve an addition problem, and Figure 2 represents a solution for subtraction problems. The model is explained below.

A. Experimenter presents a counters and the child enumerates using a preferred strategy (e.g., counting, subitization). The child is not required to name the number of counters.
B. Child encodes a counters as a set of a chunks in working memory.
C. Experimenter covers the counters so that they are hidden from the child's view. The child then relies on working memory to remember the items, so that S now consists of a units.
D. Experimenter either presents (adds) b counters to the child and inserts them into the box, or removes (subtracts) b counters from the box and presents them to the child.
E. Child encodes b as a set of b chunks in WM.
F. Child increments or decrements S in working memory b times.
G. The new S in working memory is decoded from chunks to physical counters using a counting operation.

H. Child puts down counters to represent answer.

If children do count chips and represent each counter as a chunk in working memory, then several variables in each stimulus problem could cause errors. Children might have problems quantifying, encoding, or storing a. Children might encode larger a terms (e.g., of 3 or 4) incorrectly because of counting or encoding errors. This could result because they lose track of how high they have counted as they enumerate the row of chips. Also, if working memory has a limited capacity, then the greater the magnitude of the initial S (the a term), the more working memory capacity is already taken up. Adding a b term of any size, then, might exceed the working memory storage capacity (Step G in Figure 1). Therefore, errors might occur during the calculation of the answer. This second possibility would only occur, presumably, on problems where children attempt to add as opposed to subtract b.

Children might also start to have difficulties solving the problem starting when b is presented. It is possible that larger b terms modify S beyond working memory limitations, and therefore the modified set has an increased possibility of being in error (Step G). The b term in this task is always 1 or 2 (in order to keep the problem size small), so if b is a predictor of errors, then there should be more errors on problems where b is 2 than where it is 1.

The combined effect of a and b (a+b) might also cause difficulties for the children. If children do use chunks to represent the operands of the problems, then b will be encoded regardless of whether it is being added or subtracted (Step E). If there is only one component in working memory where all numbers are stored (in Baddeley's model, this would be the spatial storage component), this combined effect might exceed capacity

on larger problems. Therefore, the value of the combined set might exceed working memory on both addition and subtraction problems.

Children might solve the problems correctly but then answer incorrectly. Accuracy might have less to do with the actual encoding of operands and calculation than with the reproduction of the answer (Step H). That is, children might have calculated the correct answer, but have trouble representing that answer with physical objects. This effect would be more apparent as the answer grows larger, because there are more counters to put down and thus more time to forget the answer. Also, on larger answers, children might have more trouble keeping track of how many counters they have already put down and therefore miscount how many more counters they must put to represent the answer.

Another predictor of accuracy might be the problem type. Addition might be easier because children simply have to continue counting up the number line to add b to a. There is some evidence that subtraction problems can be more difficult than addition problems for children (Fuson, 1986). Alternatively, with this problem set, subtraction might be easier because children never have to create large arrays when they answer subtractions problems as they do on addition problems. Therefore, if problem type is a predictor, there should be more errors on one type of problem than the other type.

The final predictor of accuracy is the Representational Set Size (RSS) of the problem. RSS is the largest number of chunks that children must hold in memory at one time. On addition problems, RSS is equivalent to a+b, and on subtraction problems RSS is equivalent to a (e.g., the RSS of the problem 3+2 is 5, whereas the RSS of the problems 3-2 is 3). If exceeding the capacity of working memory is an issue, then it

should affect accuracy independently of when it appears in the problem. Therefore, when capacity is exceeded, children tend to fail on that problem.

Thus if the problem-size effect found in young children's arithmetic is due entirely to problems in enumerating larger operands, then variables such as $\underline{a}$ or $\underline{a} + \underline{b}$ should provide optimal predictors of error rates. If the difference in difficulty between addition and subtraction is critical, then problem type should be an effective predictor. If, however, accuracy varies primarily as a function of the number of units of chunks that must be held in working memory, then RSS should be the most sensitive index of error rates.

In order to present children with two-term arithmetic problems in a way that reduces nonmathematical task demands (such as verbal demands) as much as possible, I used a technique similar to that of Levine et al. (1992) and Huttenlocher et al. (1994). I replicated Study 1 of Huttenlocher et al. (1994), and expanded on that study to explain performance errors on individual two-term problems. I then presented the children in my study with similar two-term problems. From the results of the regressions on my data, it should be possible to account for differences in problem difficulty for 4-year-olds.

Conceptual Shortcuts on Three-Term Problems

The second goal of the research was to find whether preschoolers have a conceptual understanding of the relations between numbers and operators and if they can use this understanding to create efficient solution procedures. Part of the relation between addition and subtraction is captured in the principle of inversion, that the addition and subtraction of the same quantity leaves the original quantity unchanged (i.e., $\underline{a}+\underline{b}-\underline{b}=\underline{a}$). Several researchers have studied children's understanding of the principle of inversion in

an attempt to find whether children understand the relation between addition and subtraction. Starkey and Gelman (1982) presented simple three-term problems of the form $a+b-b=a$ to 3- to 5-year-olds. Problems of this form can be solved quickly and easily by using a shortcut based on the principle of inversion. The children in Starkey and Gelman's study solved problems such as 1+1-1, and the researchers concluded that these preschoolers showed understanding of the principle of inversion based on this evidence. In Starkey and Gelman's study, however, the problems were so simple that children might easily have calculated the answers rather than using a shortcut. Bisanz and LeFevre (1990) used problems with larger operands to minimize the chance that the children were calculating the answers rather than using the inversion shortcut. Bisanz and LeFevre found that 20% of the 6-year-olds they tested showed evidence of using the inversion shortcut. Large individual differences were found between children in these studies, but evidence also exists for some consistent use of the inversion shortcut. Therefore, although it has been shown that at least some first graders are capable of using a conceptually-based shortcut, there is little compelling evidence of shortcut use in younger children. In addition to these results, I found in a previous study that some kindergarten children used the inversion shortcut at least inconsistently (Klein, 1996). Based on these results, there is evidence to suggest that young children do indeed have some conceptual understanding of inversion.

In my previous study with kindergarten students, I found that about 20% of the children used the inversion shortcut at least once in eight problems, and that another 30% of the children used a different shortcut based on the principle of associativity (i.e., that $a+b-c$ may be solved as $(a-c)+b$, thus reducing task demands). I reasoned that if

kindergarten students with little formal instruction could generate shortcut procedures to solve arithmetic problems, perhaps preschoolers also might independently generate shortcuts if the task demands were low enough. Use of a shortcut would imply that these children have a conceptual understanding of the relations between number and operators and that they can use this understanding to create efficient solution procedures. Because the children in the present study had not yet attended school, presumably they had received no formal instruction in arithmetic. Therefore, if they could add, subtract, and construct shortcuts, they probably developed these skills through their experience.

To investigate whether preschoolers can construct shortcuts, they were presented with some simple three-term problems in the second session of this study. The children attempted to solve three-term standard problems of the form $\underline{a}+\underline{b}-\underline{c}$ (e.g., 3+2-1) and inversion problems of the form $\underline{a}+\underline{b}-\underline{b}$ (e.g., 3+2-2). Performance on the standard problems was compared to performance on the inversion problems. My research with kindergarten children revealed that when children used the inversion shortcut, their solution latencies were significantly shorter than when they used counting algorithms to solve problems. Similarly, if the preschoolers use the inversion shortcut, latencies on inversion problems should be shorter than on standard problems.

Method

Participants

Forty-eight 4-year-olds (half males and half females) from five university-area preschools and daycares took part. Their median age (in yr:mo) was 4:4 (range 4:0 to 4:11). An additional six children could not complete the counting and matching pretests (described below) and thus were not included in the analyses.

Materials and Procedures

Children were interviewed individually in two 15-minute sessions on different days. We recorded each interview with a video camera positioned to record the child's face, hands, and voice. Children were told that they would be playing a number game and that they were simply to do their best.

In the first session, children completed counting and matching pretests and solved two-term problems. In the second session, children solved three-term arithmetic problems. All required the manipulation of a certain number of manipulatives (poker chips). The chips were placed on white mats to make them more visible for the video camera.

Counting pretest. Children's counting scores consisted of the highest number of chips they could count successfully. Children first counted an array of six chips. A successful count resulted in the array being increased to eight chips, and then ten. Children were given two chances to count each array, if necessary. If they counted an array unsuccessfully on the second try, the array was decreased by one (to five, seven, or nine) and the children had two more chances to count that decreased array. If they succeeded, their counting score was the magnitude of that decreased array. If they could not count the decreased array, their counting score was the magnitude of the decreased array minus one. The criterion for completion of the counting pretest was successfully counting ten chips.

Matching pretest. Children matched paired arrays of 2, 4, and 6 chips. For each array, the chips were placed on the experimenter's mat, shown to the child, and then covered with an inverted cardboard box with holes at each end large enough to insert and

remove chips. Children placed chips on their mat to match the hidden array. The matching score was the highest number to be matched successfully, and the procedure for determining the matching score was the same as that for the counting problems. The criterion for completion of the matching pretest was successfully matching six chips.

Two-term problems. Twelve two-term, single-digit problems were used (see Appendix A), half of which were addition and half subtraction. Within each type, half were small (representational set size [RSS] of two or three) and half were large (RSS of four or five). Problems were ordered unsystematically, with the constraint that half the small and half the subtraction problems appeared in the first six problems. Three orders of two-term problems were created and each was reversed, yielding six orders that then were counterbalanced across participants within sex.

Children watched while an initial array was placed on the experimenter's mat and then covered. Next, the experimenter added or subtracted a number of chips from the hidden array, ensuring that the number of chips added or removed was observed by the children. The children then attempted to put the same number of chips on their mat that the experimenter had in the box. Finally, the box was removed and the next problem was presented.

Three-term problems. Twelve problems were used, all of the form a+b-c (Appendix B). Half the three-term problems were standard, where b and c differed from each other (e.g., 1+3-2), and half the problems were inversion problems, where b and c were identical (e.g., 1+3-3).

Half the children solved standard and inversion problems in a mixed condition with the constraint that three of each problem type were in the first six problems. The

remaining children solved standard and inversion problems in blocked orders (all six of one type of problem first). Three orders of problems were constructed for both the mixed and blocked conditions, and each order was reversed to yield six orders of mixed and six orders of blocked problems. Presentation condition (mixed vs. blocked), and order were counterbalanced within sex. Two- and three-term orders were confounded across participants.[1]

At the beginning of the second session, children solved one two-term problem to remind them of the procedure. They were then instructed to observe the change in procedure for three-term problems. The a term was placed on the experimenter's mat, and the child matched it. The experimenter then covered her own mat with the box, b was added, and then c was subtracted. Children then attempted to match the experimenter's hidden array. Latencies were collected for these trials. A clock was started when the experimenter said the underlined word in the sentence "...and I take this many out" or "...and I take this many away". The clock was stopped when the experimenter removed the box to show the child the answer.

Data Coding

For two-term problems, one coder recorded accuracy data for all 48 subjects, and a second individual coded the data of 12 children to assess reliability. For three-term problems, accuracy, latency, procedures, and spontaneous verbal reports were recorded by the primary coder using a scoring manual (Appendix C). A sample of 12 subjects were coded for reliability by a second individual.

Results and Discussion

Counting and Matching Pretest

One child was eliminated from the study because he could not count to 10. Five children could not understand the matching task (i.e., they did not match any of the experimenter's arrays) and thus were also eliminated from the study. Children who did complete both the counting and matching problems replaced these children in the study.

Two-Term Problems

The reliability between the two researchers who coded the two-term problems for accuracy was 100%. We first performed an overall analysis on the two-term problems. Error data were analysed to find any differences in accuracy across different types and sizes of problems. Correlations were conducted to determine (a) which predictors were most highly related to error rate and (b) the degree to which predictors were interrelated. These predictors then were entered into paired regression analyses to find which accounted for variability in error best for the four-year-olds on arithmetic problems. Regression analyses were repeated on the data from Huttenlocher et al.'s (1994) study to find whether the same pattern of results occurred for both sets of data.

Overall Error Data

Error rates were computed for each participant and analysed with a 2(Gender) x 2(Problem Size: large vs. small) x 2(Problem Type: addition vs. subtraction) analysis of variance with repeated measures on the last two variables. The mean proportion of errors was .28. As expected, errors were more common for large problems (M = .42) than for small problems (M = .14), $F(1, 46) = 61.27$, $p < .01$. Error rates were not influenced by problem type (M = .25 for addition, .31 for subtraction) or gender (M = .24 for girls, .32

for boys), but these two variables interacted, $F(1,46) = 5.78$, $p = .02$. Tests of simple effects ($\alpha = .01$) revealed that subtraction problems were more difficult than addition problems for boys (mean error rates of .38 vs. .26) but not for girls (.24 vs. .23).

Predictors of Difficulty on Two-Term Problems

Correlational analyses. As discussed in the task analysis of the two-term problems, several possible predictors of difficulty exist on these simple arithmetic problems: a (the first operand of the problem), b (the second operand of the problem), a+b (the combined set of the first and second operands), the correct answer, problem type, and Representational Set Size (RSS). To find which variable correlated most highly with the mean error, a correlational analysis was performed including each of these predictor variables plus the criterion variable (mean error rate). The difficulty of each of the problems was estimated by averaging across the error rates of all 48 subjects for each of the 12 problems, resulting in 10 degrees of freedom for the analysis. The results of these analyses are presented in Table 1.

RSS, a+b, a, and the answer are all significantly correlated with mean error, although RSS was correlated most highly. In part because of the restricted range of questions we could present the children, however, RSS is also significantly correlated with the answer, a+b, and a. RSS, the answer, and a+b are equivalent on addition problems, and RSS and a are identical on all subtraction problems. The predictors, therefore, are not independent. Because several of the variables are highly correlated, any conclusions are qualified by the high intercorrelations among some of the predictors.

Regression analyses. A regression analysis was conducted in which RSS was entered along with all other variables (a, b, a+b, problem type, and the answer) to account

for variability in problem difficulty. The resulting $\underline{R}^2$ was .92. The roles of the individual predictors were difficult to assess because the high intercorrelations resulted in the exclusion of variables. To find whether any predictors accounted for unique variance in the dependent variable over and above that accounted for by RSS, and whether RSS accounted for any unique variance over and above the other variable, paired regressions were conducted using RSS and each of the other predictors of problem difficulty. The analyses were run with RSS entered first and the other variable entered second, and then again with these two variables entered in the opposite order (the other variable first). The results from these paired regressions are presented in Table 2. Each row of Table 2 represents results from paired regressions involving RSS and one of the other variables (labeled Other Variable in the table). The first column of data represents the $\underline{r}^2$ for RSS and the second column represents the $\underline{r}^2$ for the other variables in the analysis. The third column represents the combined $\underline{R}^2$ for the two variables (i.e., the amount of variance in mean error that the two variables account for together). The fourth column represents the unique variance for RSS in the equation, that is, the amount of variance in mean error for which RSS accounts when the effect of the other variable is partialed out. The last column represents the unique variance for the other variable in the analyses. The results show that the predictor RSS accounts for unique variance in mean error when entered with any other identified predictor of performance on the problems, whereas none of the other predictors account for any unique variance when RSS is partialed out.

From these analyses it can be concluded that, unlike the other variables, RSS accounts for unique variability in mean error above and beyond that of any other variable identified in the task analysis. Thus, RSS is the best overall predictor of difficulty on the

two-term problems. RSS is not simply an operand of the problem, but instead represents the largest number that children must keep in working memory while solving the arithmetic problem. On addition problems, this is the answer, and on subtraction problems, this is a. This suggests that the process that causes difficulties when solving two-term arithmetic problems is heavily influenced by the size of the set that children must remember.

Regression analyses of the data from Huttenlocher et. al. A regression analysis of the data from Huttenlocher et al. (1994, Study 1, Table 3), was performed to see if the same pattern of results would emerge for their data (see Table 3). Although the magnitude of unique R^2s varies somewhat, in both sets of data (a) RSS correlates more highly with error rates than any other variable and (b) the unique variability associated with RSS is significant and greater than that associated with any other variable.

The data from our study have a similar pattern as those from Huttenlocher et al. (1994), which suggests that the results may be generalizeable to more than one group of children and may indeed hold for most preschool children. Where there are large differences in the unique R^2 for RSS between the two samples, RSS accounts for more variability in Huttenlocher's et al.'s sample. Huttenlocher et al.'s data came from a group of children younger than in the more recent sample (in years:months, the children ranged from 3:9 to 3:11). It may be that younger children have even more difficulty with larger numbers that the older children, and so as RSS gets larger, it causes the younger children to make more errors than older children would. This hypothesis is supported by the pattern of results in Table 4. Because the children in Huttenlocher et al.'s (1994) study are younger than in our sample, it can also be tentatively concluded that the results may

hold for younger preschool children as well as just for four-year-olds. Regression analyses on data of the younger age groups (ranging from 2:6 to 3:8) in Huttenlocher et al's (1994) study show the same pattern of results as those of the older age groups, suggesting that the memory load and task demands contribute to errors for younger children as well as the children in the present study.

The finding that RSS is the best predictor of difficulty on arithmetic problems in preschool children can be explained with the working memory task described earlier. RSS represents the largest number that children must hold in memory at one time if they solve the problem correctly. The fact that performance drops as RSS gets larger is consistent with the hypothesis that working memory capacity is exceeded as RSS increases. Assuming that exceeding capacity results in decreased performance, children might fail because they either cannot calculate the answer properly or because they forget one of the operands of the problem.

Three-Term Problems

Overall Error and Latency Data

In order to find whether there were any effects of gender, order, or problem type on errors and latency data, error rates and latencies were computed for each participant and were analyzed with two 2(Gender) x 2(Order: Blocked vs. Mixed) x 2(Problem Type: Inversion vs. Standard) analyses of variance with repeated measures on the last two variables. Based on previous research (Klein, 1996) it was not expected that gender would have any significant effect on accuracy or latency data. It was hypothesized that children in the blocked condition might make fewer errors because each problem in each group of six was similar (and therefore more familiar) to the other questions in that

group. Finally, if children were using the inversion shortcut on inversion problems, they should solve inversion problems faster and more accurately than standard problems. The mean proportion of errors on standard problems M = (.41) did not differ statistically from that on inversion problems M = (.49). The mean latency on standard problems (8.75 s) did not differ statistically from the mean latency on inversion problems (8.63 s). There was no effect of order on either accuracy or latency. The apparent implication of these findings is that children did not solve standard problems any differently than they solved inversion problems.

Although analyses of the overall data did not reveal any patterns of results, it was clear to the observers that the children used several different procedures to solve both standard and inversion problems. Therefore, a more detailed analysis of the solution procedures was conducted.

Solution Procedures

Children used several different types of procedures to solve the three-term problems. The frequency of each procedure used was obtained, and latencies were analyzed to determine whether some procedures took longer to execute than others. Correlational analyses were conducted to find whether increased use of specific procedures on standard problems was correlated with increased use of certain procedures on the inversion problems.

Frequency of procedure use. The frequency of each of nine procedures is presented in Table 5 as a function of problem type. Two researchers coded accuracy and procedure use as described in the method section. Reliability was 92%. Two distinct groups of procedures emerged from this analysis. The first group was the two-move

procedures, where children both added and subtracted counters in order to solve the problem. The second group of procedures was apparent shortcuts, where children either added or subtracted counters but did not do both. This distinction is interesting because whereas children could solve the problems using the counters and a sequential, two-move procedure, use of a one-move procedure suggests that children were either guessing the answers or were calculating the answers in memory and then simply moving the counters to represent the answer they had in memory. It was hypothesized that children would be likely to use a two-move procedure on standard problems where they were required to calculate the answer to the problem. If children solved inversion problems using the inversion shortcut, then there should be more one-move procedures on inversion problems than on standard problems.

Procedures were coded as sequential (a two-move procedure) when children visibly added b to a and then subtracted the final term to get the answer. As shown in Table 5, children used the sequential procedure, without or with error, somewhat more often on standard than inversion problems. Although the sequential procedure was used relatively often, and with 83% accuracy overall, it was not used on the majority of standard problems, contrary to expectations. Children often did not solve the problems by simply mirroring the experimenter's actions.

Solution procedures were coded as associative when children overtly subtracted c from a and then added b. Procedures based on the principle of associativity are more efficient than the sequential procedure because subtracting first means that there are fewer chips to deal with at any one time in front of the child. For this reason, the associativity procedure is considered a shortcut, although it is still a two-move procedure.

Children who used this procedure demonstrated that they did not have to copy the experimenter in order to find the answer to the problem. Although they used the counters to help solve the problem, they found a novel procedure to do so. This finding was unexpected and supports the conclusion that preschoolers are capable of using conceptual shortcuts to solve arithmetic problems. Children used the associativity shortcut on approximately equal percentages of standard and inversion problems, and made slightly more errors on standard problems. Ambiguous two-move procedures are discussed later in this section.

The second group of procedures consisted of <u>apparent shortcuts</u>. On many of both the standard and inversion problems, children used <u>covert</u> procedures to find the answer. Procedures were coded as covert on standard problems when the experimenter asked children to answer the question and the children simply added or subtracted enough chips to or from <u>a</u> to answer the question. On inversion problems, a procedure was coded as covert when children simply advanced <u>a</u> as the answer.[2] Covert solutions on inversion problems could be inferred to be instances of the inversion shortcut. In neither case was there any overt form of calculation. Several times children solved inversion problems covertly, but behaviourally it was obvious to the experimenter that the child did not solve the problem using the inversion shortcut. These trials were recorded as Covert/Standard even though they took place on inversion problems.

On standard problems, children who used a covert procedure either must have calculated in memory as the experimenter demonstrated the problem or must have calculated very rapidly after the experimenter finished the problem. This result was unexpected. It was extremely difficult to differentiate between covert calculation on the

standard problems and inversion shortcut use on the inversion problems because children might have also been calculating covertly on inversion problems. In order to ascertain whether there were any differences between the covert computation procedure on standard problems and covert procedure use on inversion problems, several other indicators of using an inversion shortcut were examined.

Another potential indicator of inversion shortcut use is the time required to solve problems. Use of the inversion shortcut should be faster than calculating the answer to the problem. Median latencies for problems where a covert procedure was coded were analyzed with a 2(Gender) x 2(Problem Type: standard vs. inversion) x 2(Order: blocked vs. mixed) analysis of variance with repeated measures on the second variable. Only children who showed at least two instances of covert procedure use on both standard and inversion problems (four uses total) were included, resulting in a sample size of 17 children. The mean of median latencies on inversion problems (5.91 s) was significantly faster than that on standard problems (7.38 s), $F(1, 15) = 5.815$, $p < .05$. This implies that whereas children calculated answers to standard problems, they used a different, faster procedure (presumably the inversion shortcut) to solve the inversion problems. The order of the problems made no difference to the latencies; that is, children who solved problems presented in a blocked order were no faster or slower than children who solved problems presented in a mixed order. There also was no effect of gender.

To further ascertain whether there were any differences between the covert computation procedure on standard problems and covert procedure use on inversion problems, a correlational analysis was performed to determine whether the probability of using a covert calculation procedure on a standard problem correlated with a higher

probability of using a covert procedure on an inversion problem. A significant positive correlation between use of a covert calculation procedure on standard problems and use of a covert procedure on inversion problems might indicate that children were using the same covert procedure on both types of problems. A nonsignificant result, however, might suggest that the covert procedure used on the standard problems was a different procedure than that used on inversion problems, and that that different children used the different covert procedures. The correlation between use of covert procedures on standard and inversion problems $\underline{r}(48) = .21$, $p > .05$, suggesting that there was no significant relation between using a covert calculation procedure on a standard problem and using a covert procedure on an inversion problem.

The pattern of results in Table 6, however, indicates that most children who used a covert procedure at least once on a standard problem also used a covert procedure at least once on an inversion problem. There are at least two possible explanations for this pattern. First, children who use a covert calculation procedure on standard problems may use the same covert calculation procedure on inversion problems. Although this is a logical conclusion, latency and verbal report data (presented later) do not support the argument. The second explanation is that the children who use a covert calculation procedure on the standard problems are more comfortable with arithmetic in general and so are more likely than their counterparts who must use counters to calculate the answers to try other shortcuts (like the inversion shortcut) to solve arithmetic problems when the opportunity arises. No firm conclusions can be made from the data in this study, but taken together the correlational, latency, and verbal report data suggest that at least some children are using both a covert calculation procedure and the inversion shortcut.

Although we did not require any verbal reports, we noted any spontaneous reports that occurred. On about 30% of the trials where children solved inversion problems with a covert procedure, they spontaneously reported that $\underline{b}$ and $\underline{c}$ were the same, so they didn't count, or words to that effect. For example, one child reported that "they're the same, I didn't do anything." This implies that the children did not calculate answers to those inversion problems, but instead used the inversion shortcut.

Taken together, the above findings suggest that children are solving some inversion problems covertly, but with a different procedure than covert calculation. Therefore, several tentative conclusions can be drawn. First, at least some preschoolers have some conceptual understanding of the relation between addition and subtraction. That is, they must understand that subtraction is the inverse of addition, and so if one adds and subtracts an identical number to a set, then there is no change in the original set. Because it is doubtful that preschoolers have been taught this relation, we can conclude that even at this age children are capable of discovering new solution strategies by themselves by using their conceptual understanding of addition and subtraction.

Second, as well as understanding the relation between addition and subtraction, children who use the inversion shortcut probably understand that it is easier to use the shortcut than to calculate all of the steps. This implies that those children do not simply follow a set of rules to solve problems, but instead use their understanding to solve problems quickly and easily. Using the inversion shortcut requires paying attention to all three elements of the problem simultaneously, rather than solving the problem using a left-to-right procedure. Thus children using the inversion shortcut understand the

problem, know the goal state, and look for the least effortful way to solve the problem. These skills are also unlikely to be taught at this age.

The fact that the children are also using covert computational procedures should not be overlooked, however. It was not expected that these children would solve standard problems without the use of manipulatives, but they demonstrated convincingly that they are capable of solving three-term arithmetic problems with minimal use of tangible aids.

Occasionally children solved a standard problem by stating that the answer did not change. Thus, they may have overextended the inversion shortcut and used it to answer standard problems as well as inversion problems. This hypothesis is supported by the fact that in all but one of the seventeen cases, the children who overextended the inversion shortcut also used a covert procedure on inversion problems. Children who show an overextension of inversion must not fully understand the concept of inversion because otherwise they would know that they can not use the procedure when b and c are unequal.

The negation procedure occurred either when children began to add b to a (Sequential/Negation) or began to subtract c from a (Associativity/Negation) on inversion problems and then stopped, and behaviourally indicated that they knew that they did not need to calculate to find the answer because "it didn't change." Children who use a negation procedure are likely showing some evidence of understanding the concept of inversion. Negation occurred once on a standard problem, where the child indicated to the experimenter that the value did not change (this was, of course, in error). Negation happened infrequently but suggests that the children did realize that inversion problems could be solved without calculating the answer by using the inversion shortcut.

Although children used many interesting and clever solutions to solve the arithmetic problems, they also made many errors. Simple computational errors (i.e., putting down four counters instead of three when solving a problem using a sequential procedure) were included in the number of occurrences for that procedure. Several other types of errors were also noted, however. The errors also could be grouped into 2-move and apparent shortcut categories. In Table 5, the 2-move errors are collapsed into one group, but three different types of errors occurred, all with low incidences, and are described below.

The 2-move ambiguous – switched signs procedure occurred when children subtracted b and then added c (that is, they switched the order of the operators). For example, on the problem 3+1-2, the child subtracted 1 and then added 2 (3-1+2).

The procedure 2-move ambiguous – switched operands was coded when children added c and then subtracted b (i.e., they switched the order of the operands). For example, on the problem 3+1-2, the child added 2 and then subtracted 1 (3+2-1).

Some children both added and subtracted, but it was not clear what they were trying to do. This was coded as 2 move ambiguous – 2 incorrect. Neither the number they added nor the number they subtracted was equal to one of the operands.

The second major category of errors consisted of procedures on which children used only one move (either addition, subtraction, or saying "no change") to arrive at the answer to the problem.

On some problems, children missed adding or subtracting one of the terms and thus ended up calculating a+b or a-c instead of the whole problem. This was coded as omits operation. Almost all the children omitted an operation on at least one problem in

the set, and this occurred more often on standard problems (82 times) than on inversion problems (64 times). The children might possibly have been attempting to find more efficient ways to solve the standard problems and accidentally used a faulty shortcut procedure. Another possibility is that children were trying to go faster on and missed a step in the problem while trying to increase their speed.

Finally, sometimes children either added or removed some number of chips that did not correspond to any term in the problem. This was coded as 1-move ambiguous. In this

case, it could not be determined whether children knew the answer and simply put down the wrong number of chips (i.e., they were executing a covert procedure and made an error), or whether they were just guessing at the answer. Most times (85%) the deviation was 1 away from *b* or *c*. There was a large number of 1-move ambiguous errors on the inversion problems. Although the reasons for this are unclear, it is possible that children were aware that there was something special about the inversion problems and were trying to find a solution that was quicker and easier than calculating the answer. Because it was not possible to tell how the children solved the problem, no further analysis of these errors was performed.

General Discussion and Conclusion

The purpose of the study was twofold: to investigate what characteristics make two term-problems difficult for preschoolers, and to investigate whether preschoolers can use conceptual shortcuts to solve novel three-term arithmetic problems. Simple two-term problems were presented to four-year-olds using a nonverbal method in order to alleviate some of the task demands, and the children's performance on those problems was

recorded. The children's pattern of errors could be explained using a model of working memory, suggesting that one of the reasons that children have difficulties solving two-term problems is that their working memory is not efficient enough to solve the problems easily. A similar nonverbal task was used to present three-term problems to the children to investigate how they would solve novel arithmetic problems where a conceptual shortcut could be used to solve the problem quickly and easily. If the children had a conceptual understanding of the relation between addition and subtraction then they should have been able to create conceptual shortcuts to solve the problems. The fact that the children did use conceptual shortcuts supports Starkey and Gelman's (1982) claim that preschoolers have some conceptual understanding of the relation between addition and subtraction.

The children in this study were all capable of solving small two-term problems. They solved two-term problems best when the RSS was smaller (2 or 3 vs. 4 or 5). The best predictor of accuracy was RSS, which represents the largest number that must be kept in working memory if the problem is solved with a valid procedure. RSS is the same as $\underline{a+b}$ on addition problems and as $\underline{a}$ on subtraction problems. Because accuracy decreases as RSS increases, it is likely that larger values of RSS exceed working memory capacity and cause failures. This suggests that one major cause of difficulties on two-term problems is that WM is not efficient enough allow the children to solve the problems easily.

As children mature and working memory develops, several changes may occur. Working memory storage may increase as storage strategies become more efficient (Case, Kurland, & Goldberg, 1982). Also, the central executive control might become

more efficient, allowing working memory to process problems more quickly and accurately. The results of this study have implications for the understanding of the development of arithmetic skills in young children. It seems that adding and subtracting by themselves are not so much a problem for young children as is keeping the operands of the problems in memory. That is, the children understand the concepts of addition and subtraction, but they are incapable of arriving at a correct answer when they have to keep a larger number in memory. Teachers presenting new mathematical concepts to children (both preschool and school-aged) might lower the task demands of learning the new skill by ensuring that the children do not have to keep many chunks of information in working memory while they are learning to solve the problems.

The children in this study solved three-term problems using a variety of procedures. Most children used a covert computational procedure at least once, and many children also showed use of shortcuts such as associativity and inversion. Both of these shortcuts require an understanding of some underlying conceptual principles of arithmetic. We can be fairly sure that the children have not been taught to use these shortcuts because they have not yet entered the school system.

When solving complex mathematical problems, it is advantageous to have efficient procedures to solve the simple problems that are part of the larger problem. Efficiency can be indexed by speed and accuracy. The inversion shortcut that many children used in this study was faster and more accurate than both the sequential and covert procedures.

The high frequency of conceptual errors such as Omits Operation could be attributed to several causes. The children may have made these errors because they did

not understand the task and/or arithmetic. Alternatively, these errors might have occurred because children were trying to solve the problems as quickly as possible, and sometimes missed steps of the problem while trying to do so. The fact that children used this procedure more on standard problems than on inversion problems might be because the children were looking for quicker ways to solve standard problems, whereas they may have already found a quicker way to solve inversion problems (the inversion shortcut) Some of the procedures that the children used were shortcut procedures that were based on principles of arithmetic. The inversion shortcut, the associativity shortcut, and negation were all procedures that children could only use if they had some understanding of the principles of addition and subtraction. We can conclude that some of the preschoolers did have a conceptual understanding or arithmetic, and that they probably did not learn these shortcuts in school. Presumably, all the children will learn more about the constraints of arithmetic and therefore make fewer conceptual errors as they grow older.

Because we did not ask children how they solved each three-term problem, we were limited in our ability to conclude which procedures they used to solve the inversion problems. The evidence outlined above leads to the conclusion that children did use the inversion shortcut, but we cannot be certain. Probably the most certain method of discovering how children solved each problem would be to ask them to provide retrospective verbal reports as they solve the problems.

The preschoolers in this study showed more understanding of mathematical concepts such as inversion and associativity than have been demonstrated before. The nonverbal method used in this study might have alleviated some of the task demands,

thereby allowing the children to concentrate on the problems rather than struggling with the language. Using nonverbal methods on other tasks might allow preschoolers to show how much they really know about underlying concepts and change our understanding of the child's developing mind.

Overall the children in this study showed a remarkable understanding of simple arithmetic. Even on three-term problems, which we assume are novel to them, the children solved the problems quickly and used a variety of different solutions that did not require the use of manipulatives. In this study no correlations were analyzed between accuracy on two-term problems and that on three-term problems. Children who solve the two-term problems more accurately might be more likely to solve the three-term problems more accurately assuming that success on two-term problems implies a good understanding of mathematical concepts, but other relations are less easy to predict. Perhaps children who perform best (i.e., fast latencies and high accuracy) on two-term problems would be less likely to use conceptual shortcuts on three-term problems simply because they can use covert procedures so easily. Conversely, the children who perform best on two-term problems may do so because their working memory can handle larger numbers. If so, those children might be more likely to discover the inversion shortcut because they can hold more numbers in memory at once, and therefore see the relations between those numbers (i.e., b and c are the same).

This research revealed new information about how proficient 4-year-olds can be at solving simple arithmetic problems, and that even before they start formal schooling, they are already beginning to understand the relations between addition and subtraction. The research also revealed that one of the main constraints on arithmetic performance for

this age group might be the relatively small capacity of their working memory. If this is so, and if, as some researchers suggest, memory processes improve and become more efficient with practice, then as working memory capacity increases as children practice, a major constraint on arithmetic performance will lessen and children will be able to solve more difficult arithmetic problems.

Footnotes

[1]One other between-subjects condition was used in this experiment. Children in the set-undefined condition were told "I put this many in and take this many away" on all problems. Children in the set-defined condition, however, heard "I put this many in and take the same number away again" on the inversion problems. When the tapes were coded, however, it was noticed that the experimenter was extremely inconsistent in the use of the appropriate wording. No differences in accuracy were found between the set-defined and set-undefined conditions, and this between-subject variable was removed from all subsequent analyses.

[2]On five occasions, children, when prompted to solve an inversion problem, removed all the a-term counters and replaced them with the same number of counters from their pile of counters. This probably occurred because children felt they had to make some change to the counters in front of them but knew that the answer was same as the number of counters they already had. These occurrences were included in the total number of covert procedures on inversion problems.

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Appendix A

Two-term Problems Presented to Students

Addition Problems	Subtraction Problems
1+1	2-1
1+2	3-1
2+1	3-2
3+1	4-1
3+2	5-1
4+1	5-2

Appendix B

Three-term Problems Presented to Students

Standard Problems	Inversion Problems
1+3-2	1+3-3
2+2-1	2+1-1
3+1-2	3+1-1
3+2-1	3+2-2
4+1-2	4+1-1
4+2-1	4+2-2

Appendix C

Coding Procedure for Three-Term Problems From Videotape

1. Before starting the videotape, write the problems (found in the testing booklet for each child) in the Problems column.
2. Record the child's answer in the Answer column.
3. Time the trial and record the latency in the Latency column.
   - Starting the clock:
     - When removing c, the experimenter says one of a few things:
       1. "and I take this (or these) many away"
       2. "and I take this many out"
     - Start the clock when the experimenter says the underlined word
   - Stop the clock at the moment when the experimenter removes the box to show the child the answer.
4. Describe what the child does in the Description column.
   - If the child simply adds and/or subtracts chips, record the operation and the number added/subtracted.

     E.g., If the child adds 2 and then subtracts 3, record "+2-3"
   - If the child holds the chips in his/her hand, but doesn't place any on the mat (common on inversion problems), indicate how many the child held, and that he/she didn't put the chips on the mat.

     E.g., If the child holds 3 in his/her hand and then places those chips back on the pile instead of on the mat, record as "held 3, none on mat".

- If the child anticipates the experimenter by picking up b before the experimenter finishes the problem, and then puts b down on the mat when the experimenter finishes, record this as AM (Anticipatory Movement), record the number of chips that the child picked up, and record in brackets that this number corresponds to b.

  E.g., If the experimenter shows adding a b term of 2 and the child picks up the 2 chips before the problem is finished, record as AM 2(b).

- If the child does nothing, write "no change".

5. Record the procedure in the Solution Procedures column (see Table C1 for codes).

Finally, record anything the child says that is relevant to solving the problem in the Verbal/Observations/Comments column.

Table C1

Codes for Solution Procedures for Three-Term Problems

Code	Description
Sequential (SQ)	Child places b on mat and subtracts c.
SQ with Error (SQ/e)	Child makes an error while adding b or subtracting c.
Associativity (Assoc)	Child subtracts c and then adds b.
Associativity with Error (Assoc/e)	Child makes an error while subtracting c or adding b.
Omits Operation (OO)	Child either adds b or subtracts c, but not both.
Covert	On standard problem, child simply adds/subtracts enough chips to get an answer without any overt calculation.
Inversion (INV)	On an inversion problem, child does not touch chips, and simply answers by saying "it doesn't change.
Overextend INV (OINV)	On a standard problem, the child says nothing changes.
SQ/NEG	Child adds chips to the mat and then indicates (verbally or behaviourally through shaking head, etc) that he/she did not need to do so, and quickly removes chips.
Assoc/NEG	Child removes chips from the mat and then indicates (verbally or behaviourally through shaking head, etc) that he/she did not need to do so, and quickly replaces chips.

Code	Description
One Move Ambiguous (1MA)	Child either adds or removes some number of chips that is not consistent with b or c and does not result in the correct answer
Two Move Ambiguous – Switched Signs (2MA/SS)	Child subtracts b and then adds c. E.g., on the problem 3+2-1 the child calculates 3-2+1.
Two Move Ambiguous – Switched Operands (2MA/SO)	Child adds c and then subtracts b. E.g., on the problem 3+2-1, the child calculates 3+1-2.
Two Move Ambiguous – 2 Incorrect (2MA/2)	Child adds and subtracts while calculating the answer, but both operands differ by at least two from the correct term.

Table 1

Correlations Between Potential Predictors of Problem Difficulty

Variable	a	b	a+b	Answer	Problem type	RSS
Error rate	.867**	.239	.897**	.655*	.169	.940**
a		.000	.939**	.400	.516	.817**
b			.343	.000	.000	.213
a+b				.376	.485	.841**
Answer					-.516	.817**
Problem type						.000

** $p<.01$, *$p<.05$, $df=10$

Table 2

Results of Regressions Involving RSS and Other Potential Predictors of Problem Difficulty

Other Variable	R^2			Unique R^2	
	RSS	Other Variable	Combined	RSS	Other
a	.88**	.75**	.91**	.16**	.03
b	.88**	.06	.89**	.83**	.00
a+b	.88**	.80**	.92**	.12**	.04
Answer	.88**	.43*	.92**	.49**	.04
Problem type	.88**	.03	.91**	.88**	.03

Note. The column labeled "Other Variable" refers to the variable that was entered into the regression with RSS. The first, second, and third columns of data refer to the R^2 for RSS, the other variable in the equation, and the two variables combined, respectively. The fourth and fifth columns refer to the unique R^2 for each variable when the other variable in the analysis is partialed out.

*$p<.05$, **$p<.01$.

Table 3

Results of Regressions Involving RSS and Other Potential Predictors of Problem Difficulty [Using Data[a] From Huttenlocher et al. (1994, Table 3, ages 3:9 to 3:11)]

Other Variable	R^2			Unique R^2	
	RSS	Other Variable	Combined	RSS	Other
a	.78**	.29	.78**	.50**	.00
b	.78**	.07	.78**	.72**	.00
a+b	.78**	.40*	.79**	.39**	.00
Answer	.78**	.32	.79**	.47**	.00
Problem type	.78**	.00	.82**	.82**	.04

Note. The Other Variable column refers to the variable that was entered into the regression with RSS. The first, second, and third columns of data refer to the R^2 for RSS, the other variable in the equation, and the two variables combined, respectively. The fourth and fifth columns refer to the unique R^2 for each variable when the other variable in the analysis is partialed out. The regressions show that RSS always accounts for unique variance over and above that accounted for by the combination of RSS and the other variable.

[a]n=30 children

*p<.05,**p<.01.

Table 4

Comparison of Percent Mean Errors From Present Study and From Huttenlocher et al. (1994, Table 3, ages 3:9 to 3:11).

Problem Type	RSS	Klein's Data[a]	Huttenlocher's Data[b]
Addition	2	4.1667	3
	3	17.7083	38.5
	4	27.0833	63.3
	5	41.6667	70
Subtraction	2	12.5000	30
	3	14.5833	41.5
	4	33.3333	65
	5	54.1667	N/A

Note. The two data sets differed slightly on the number of problems that constituted each RSS set.

[a]n=48. [b]n=30

Table 5

Frequency and Error Rates for Solution Procedures

		Standard Problems		Inversion Problems		All Problems	
	Number of Children	% of trials	% errors	% of trials	% errors	% of trials	% errors
Two-move Procedures							
Sequential	19	19	20	13	12	16	17
Associativity	14	5	23	5	14	5	18
Ambiguous	10	4	100	1	100	2	100
Apparent Shortcuts							
Covert/Standard	33	32	-	1.7	-	17	-
Covert/Inversion	38	-	-	39	-	20	-
Overextended Inversion	12	6	100	-	-	3	-
Sequential Negation	6	.3	.3	2	-	1	-
Associative Negation	2	0	-	.6	-	.3	-
Ambiguous	29	6	100	16	100	11	100
Operation Omitted	45	28	100	22	100	25	100

Note. The total number of children is 48, and the relative frequency is based on 288 problems of each type (48 children x 6 problems). The symbol "-" is used to denote cells that are undefined (e.g., where an error is impossible because the procedure is defined only for correct uses).

Table 6

Number of Children Using a Covert Computation Procedure at Least Once on Inversion Problems vs. on Standard Problems

		Inversion	
		No	Yes
	No	5	10
Standard	Yes	5	28

Figure 1. Proposed model for how children might solve addition problems (a + b)

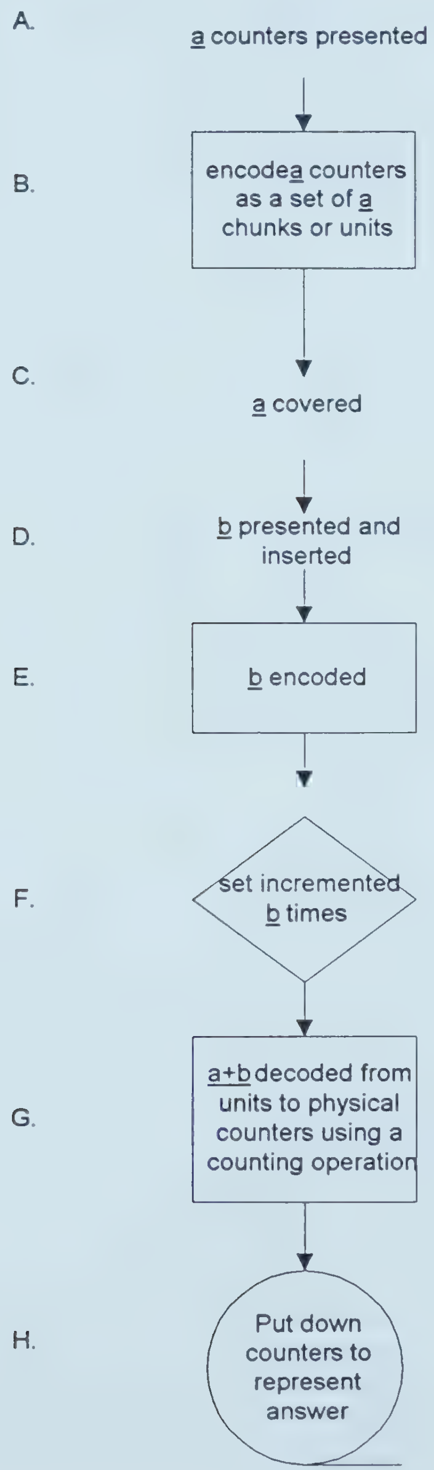

Figure 2. Proposed model for how children might solve subtraction problems (a – b)